## XIII. Parallel 1st-Order Reactions:

A. Observed:

A famous physical organic chemist made the following observation...


When the rate of formation of the products were monitored independently, it was found that...

$$
k_{\mathrm{H}} / k_{\mathrm{D}}=2.1 \text { in both cases!! }
$$

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B. Given:

$$
\begin{aligned}
& \mathrm{A} \xrightarrow{k_{1}} \mathrm{P}_{1} \\
& \mathrm{~A} \xrightarrow{k_{2}} \mathrm{P}_{2}
\end{aligned}
$$

- This case is central to the control of selectivity, but it can be deceptive.
C. Rate Equation:

1. Species A:

$$
\begin{aligned}
& -d[\mathrm{~A}] / d \mathrm{t}=k_{1}[\mathrm{~A}]+k_{2}[\mathrm{~A}]=\left(k_{1}+k_{2}\right)[\mathrm{A}] \\
& {[\mathrm{A}]=\left[\mathrm{A}_{0}\right] \mathrm{e}^{-\left(k_{1}+k_{2}\right) \mathrm{t}}}
\end{aligned}
$$

2. Species $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ :

$$
d\left[\mathrm{P}_{1}\right] / d \mathrm{t}=k_{1}[\mathrm{~A}]=k_{1}\left[\mathrm{~A}_{0}\right] \mathrm{e}^{-\left(k_{1}+k_{2}\right) t}
$$

Integrate... (CRC \#12)

$$
\begin{equation*}
\left[\mathrm{P}_{1}\right]=\frac{k_{1}\left[\mathrm{~A}_{\mathrm{o}}\right]}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right) \tag{1}
\end{equation*}
$$

Similarly ...

$$
\begin{equation*}
\left[P_{2}\right]=\frac{k_{2}\left[\mathrm{~A}_{\mathrm{o}}\right]}{k_{1}+k_{2}}\left(1-\mathrm{e}^{-\left(k_{1}+k_{2}\right) t}\right) \tag{2}
\end{equation*}
$$

D. Graphics:


$$
\begin{array}{|}
\mathrm{A} \xrightarrow{k_{1}} \mathrm{P}_{1} \\
\mathrm{~A} \xrightarrow{k_{2}} \mathrm{P}_{2}
\end{array}
$$

1. Starting Material A :

Let...

$$
\mathrm{f}(\mathrm{x})=a \mathrm{e}^{b \mathrm{x}}
$$

such that...

$$
\mathrm{f}(\mathrm{x})=[\mathrm{A}] \quad \mathrm{x}=\mathrm{t} \quad a=\left[\mathrm{A}_{0}\right] \quad b=-\left(k_{1}+k_{2}\right)
$$

2. Products $\mathrm{P}_{\underline{1}}$ and $\mathrm{P}_{\underline{2}}$ :

Let...

$$
\mathrm{f}(\mathrm{x})=a\left(1-\mathrm{e}^{b \mathrm{x}}\right)
$$

such that...

$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=\left[\mathrm{P}_{1}\right] \text { or }\left[\mathrm{P}_{2}\right] \\
& a=k_{1}\left[\mathrm{~A}_{0}\right] /\left(k_{1}+k_{2}\right) \\
& \left(\text { or } k_{2}\left[\mathrm{~A}_{0}\right] /\left(k_{1}+k_{2}\right)\right) \\
& b=-\left(k_{1}+k_{2}\right)
\end{aligned}
$$

- The values of $k_{1}$ and $k_{2}$ can be extracted from the values of adjustable parameters $\underline{\mathrm{a}}$ and $\underline{\mathrm{b}}$. However...


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## E. Caveat:

Recall, for a 1st-order reaction...

$$
\begin{aligned}
& {[\mathrm{P}]=\left[\mathrm{A}_{0}\right]-[\mathrm{A}]=\left[\mathrm{A}_{0}\right]-\left[\mathrm{A}_{0}\right] \mathrm{e}^{-k t}=\left[\mathrm{P}_{\infty}\right]\left(1-\mathrm{e}^{-\mathrm{kt}}\right)} \\
& \mathrm{f}(\mathrm{x})=a\left(1-\mathrm{e}^{b \mathrm{x}}\right)
\end{aligned}
$$

- Therefore, formation of the products in parallel 1st-order reactions follow the same mathematical form as a single 1st-order reaction. However, in a parallel reaction, $b=-\left(k_{1}+k_{2}\right)$. The difference appears in the pre-exponential term, $\underline{a}$.
- If you monitor the formation of either $\mathrm{P}_{1}$ or $\mathrm{P}_{2}$ and analyze the data as a simple first-order reaction, you will get the same rate constant!
- If either $k_{1}$ or $k_{2}$ has a kinetic isotope effect, the rate of formation of both products will show a kinetic isotope effect. Moreover, formation of $\underline{P}_{1}$ and $\underline{P}_{2}$ will exhibit the same kinetic isotope effect if treated as a simple (rather than parallel) 1 st-order reaction! Mechanistic branch points can be deceptive, and an undetected parallel pathway could be very misleading.
F. Selectivity:

$$
\begin{array}{|ll|}
\hline \mathrm{A} \xrightarrow{k_{1}} & \mathrm{P}_{1} \\
\mathrm{~A} \xrightarrow{k_{2}} & \mathrm{P}_{2} \\
\hline
\end{array}
$$

By dividing through the integral forms of the rate equations (eq (1) and eq (2))...

$$
\left[\mathrm{P}_{1}\right] /\left[\mathrm{P}_{2}\right]=k_{1} / k_{2}
$$

Alternatively, the integration can be skipped...

$$
\left[\mathrm{P}_{1}\right] /\left[\mathrm{P}_{2}\right]=\frac{d\left[\mathrm{P}_{1}\right] / d \mathrm{t}}{d\left[\mathrm{P}_{2}\right] / d \mathrm{t}}=\frac{k_{1}[\mathrm{~A}]}{k_{2}[\mathrm{~A}]}=\frac{k_{1}}{k_{2}}
$$

